North Sydney Girls High School

2023 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 9 – 44)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

NAME:_____ **TEACHER:**_____

STUDENT NUMBER:

Question	1–10	11–21	22–29	30-34	Total
Mark	/10	/30	/31	/29	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

---

1 What is the value of $\sqrt{\frac{9.5^2}{e^{-3.8}}}$ to three significant figures?

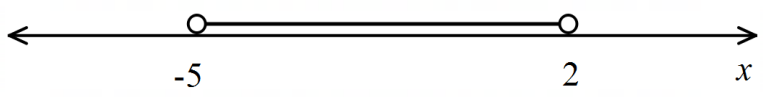
A. 1.42

B. 13.5

C. 63.5

D. 424

2 Which quadratic inequation gives the solution set shown below?



A. $(x-2)(x-5)>0$

B. $(x-2)(x-5)<0$

C. $(x-2)(x+5)>0$

D. $(x-2)(x+5)<0$

3 Let $f(x)=\dfrac{2}{3-x}$ and $g(x)=x-5$. What is the domain of $f(g(x))$?

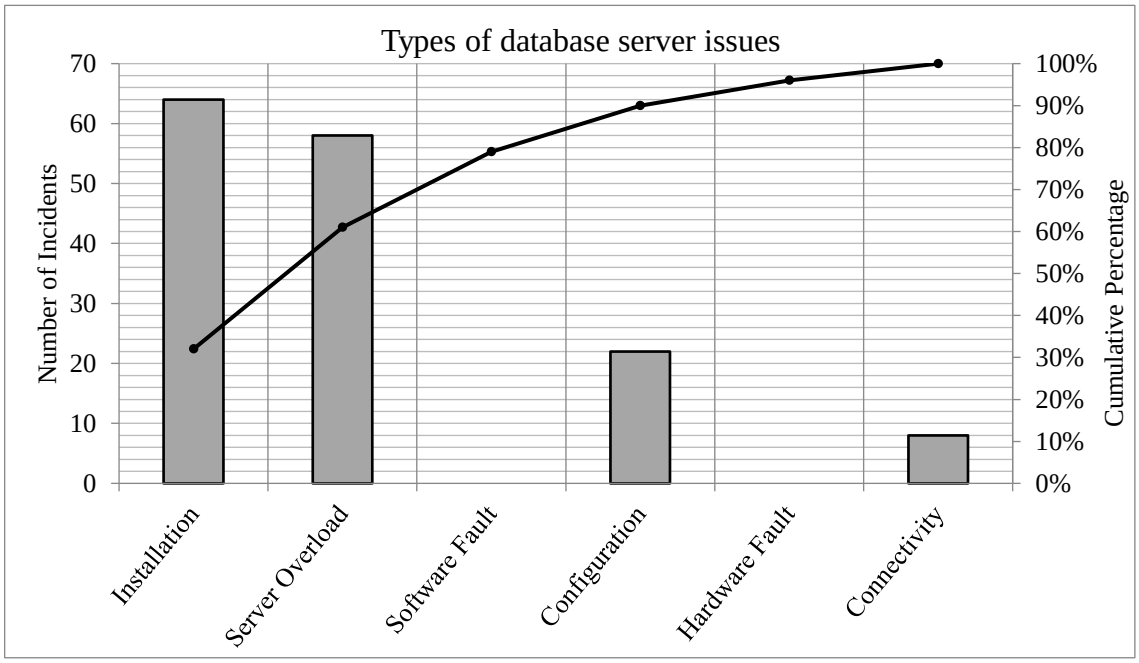
A. $(-\infty,\infty)$

B. $(-\infty,2)\cup(2,\infty)$

C. $(-\infty,3)\cup(3,\infty)$

D. $(-\infty,8)\cup(8,\infty)$

4 The following Pareto chart shows 200 incidents from a report of database server issues. The data for 'Software Fault' and 'Hardware Fault' is missing.



How many incident reports related to the issue of 'Software Fault'?

A. 24 incidents

B. 36 incidents

C. 58 incidents

D. 148 incidents

5 A bus company runs tours around North Sydney. Each tour costs the company $273 in fuel plus $48 per person in other costs. Tourists are charged $65 per person for the tour.

What is the minimum number of tourists the bus company needs to recruit for the tour to break even?

A. 3 tourists

B. 5 tourists

C. 16 tourists

D. 17 tourists

6 A discrete random variable X has the following probability distribution:

x	k	1	3
$P(X = x)$	p	0.4	$2p$

What is the value of k such that $E(X) = 0$?

A. -8

B. -3

C. 0

D. 0.3

7 The cohort mean and standard deviation for an assessment task of each course is provided in the table below. The results of a particular student are also listed in the last column.

Course Name	Mean	Standard Deviation	Student Mark
English	71	6.3	76
Mathematics	68	5.8	72
Legal Studies	32	1.4	34
Chemistry	24	2.7	22

Which subject did the student perform better in compared to their respective means?

A. English

B. Mathematics

C. Legal Studies

D. Chemistry

8 What is the primitive function of $x + \sec^2 x$?

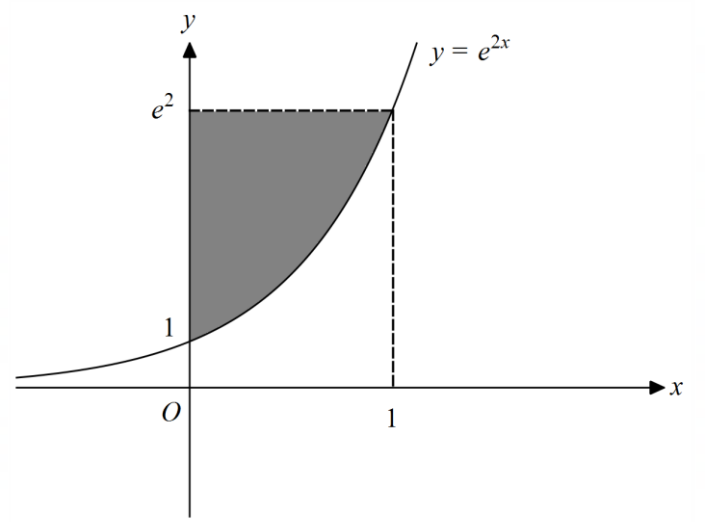
A. $\frac{x^2}{2} + \frac{1}{3}\sec^3(x) + c$

B. $x + \frac{1}{2}\sec(x) + c$

C. $\frac{x^2}{2} + \tan(x) + c$

D. $\frac{x^2}{2} + \frac{1}{2}\tan(x) + c$

9 Four different students proposed the following calculations to find the area of the shaded region in the diagram below.



Student 1: $\int_0^1 e^{2x}\,dx$

Student 2: $e^2 - \int_0^1 e^{2x}\,dx$

Student 3: $\int_1^{e^2} e^{2y}\,dx$

Student 4: $\int_0^{e^2} \frac{\log_e y}{2}\,dy$

Which student(s) is correct?

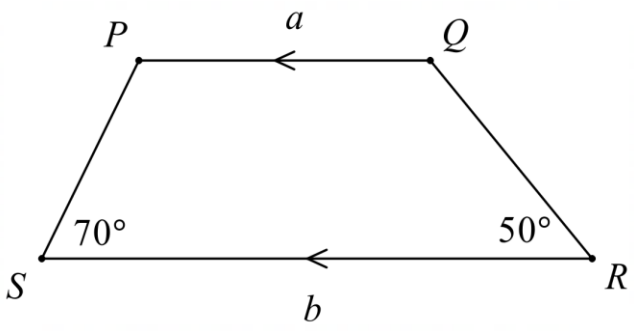
A. Student 2 only

B. Students 2 and 3 only

C. Students 1 and 4 only

D. Students 2 and 4 only

10 In the figure below, $PQ \parallel RS$, $PQ = a$ and $RS = b$. What is the length of QR?



A. $\dfrac{(b+a)\sin 70^\circ}{\sin 60^\circ}$

B. $\dfrac{(b+a)\sin 70^\circ}{\sin 50^\circ}$

C. $\dfrac{(b-a)\sin 70^\circ}{\sin 60^\circ}$

D. $\dfrac{(b-a)\sin 70^\circ}{\sin 50^\circ}$

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Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–34
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–21 (30 marks)
Booklet 2 – Attempt Questions 22–29 (31 marks)
Booklet 3 – Attempt Questions 30–34 (29 marks)

Instructions

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Please turn over

Question 11 (2 marks)

Solve $|5-x|=3$. **2**

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Question 12 (2 marks)

If $\sin A=\frac{12}{13}$ and A is obtuse, find the exact value of $\cot A$. **2**

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Question 13 (2 marks)

Find the values of m if the equation $4x^2+(m+3)x+9=0$ has two equal roots. **2**

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Question 14 (3 marks)

It is known that the fourth and ninth terms of an arithmetic sequence are 48 and 1925 respectively.

(a) Find the common difference and the first term of the sequence. **2**

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(b) Find the sum of the first 10 terms of the sequence. **1**

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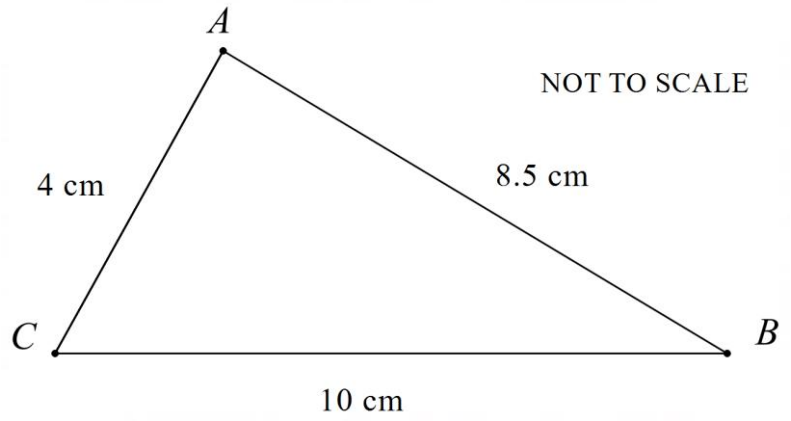
Question 15 (2 marks)



In the diagram above, ABC is a triangle with $AB = 8.5$ cm, $AC = 4$ cm and $BC = 10$ cm. **2**

Determine the size of $\angle ACB$, correct to the nearest degree.

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Question 16 (3 marks)

Find the equation of the tangent to the curve $y = x^2 e^{3x}$ at the point where $x = \frac{1}{3}$. **3**

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Question 17 (3 marks)

Show that $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$. **3**

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Question 18 (4 marks)

The number of birds, N, in a colony at time t years after observation begins is represented by the model $N = 100 + Ae^{-0.1t}$. Initially, there were 400 birds in the colony.

(a) Find the value of A. **1**

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(b) The colony is considered inactive when the number of birds fall below 125. **2**

After how many years will the colony be inactive? Give your answer to 2 decimal places.

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(c) What is the eventual population of the colony? **1**

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Question 19 (3 marks)

Solve $2\sin^2 x - \sin x = 1$ for $x \in [0,\ 2\pi]$ **3**

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Question 20 (2 marks)

Find $\int x^2\, 2^{x^3}\, dx$. **2**

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Question 21 (4 marks)

Let $y = \ln\left(\frac{4-x}{4+x}\right)$.

(a) Show that $\frac{dy}{dx} = \frac{8}{x^2 - 16}$ **2**

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(b) Hence, evaluate $\int_{-3}^{3} \frac{24}{x^2 - 16}\, dx$, giving your answer as an exact value. **2**

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End of Section I

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Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 22–29 (31 marks)

Instructions

- Write your student number at the top of this page.
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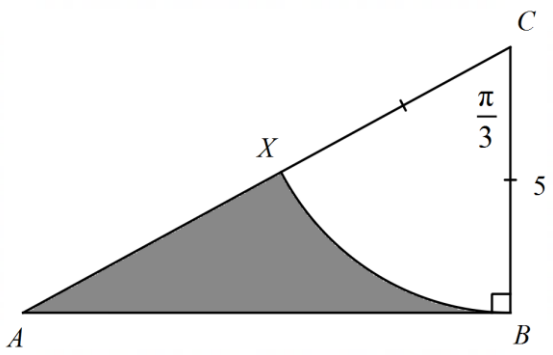
Please turn over

Question 22 (3 marks)

In ΔABC, $\angle ABC = \frac{\pi}{2}$, $\angle ACB = \frac{\pi}{3}$ and $BC = 5$ units. **3**

A circular arc, of centre C and radius CB, cuts the side CA at X.



Find the exact area of the shaded region ABX.

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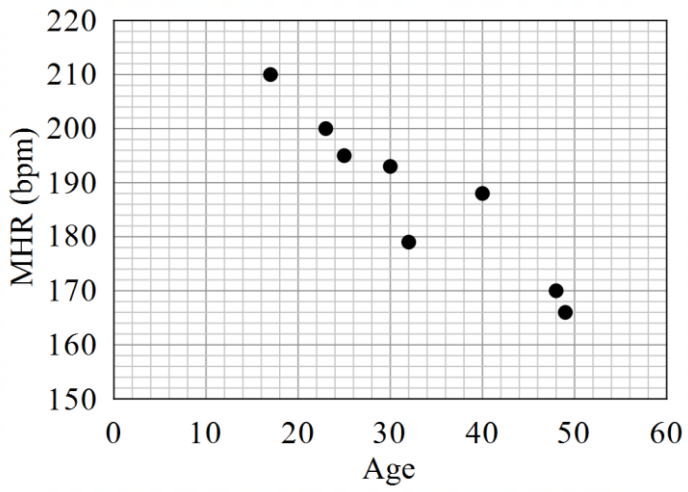
Question 23 (4 marks)

An anonymous survey was conducted at a local gym, which collected information about the respondents' age and perceived maximum heart rate (MHR) measured in beats per minute (bpm). A random sample of eight responses were selected and shown in the table and scatterplot below.

Response	A	B	C	D	E	F	G	H
Age	17	23	25	30	32	40	48	49
MHR (bpm)	210	200	195	193	180	185	170	169



(a) Find the correlation coefficient and describe the association between the age and the maximum heart rate. **2**

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(b) Find the equation of the least squares regression line using your calculator. **2**
Use the equation to predict the MHR of a 36-year old respondent.

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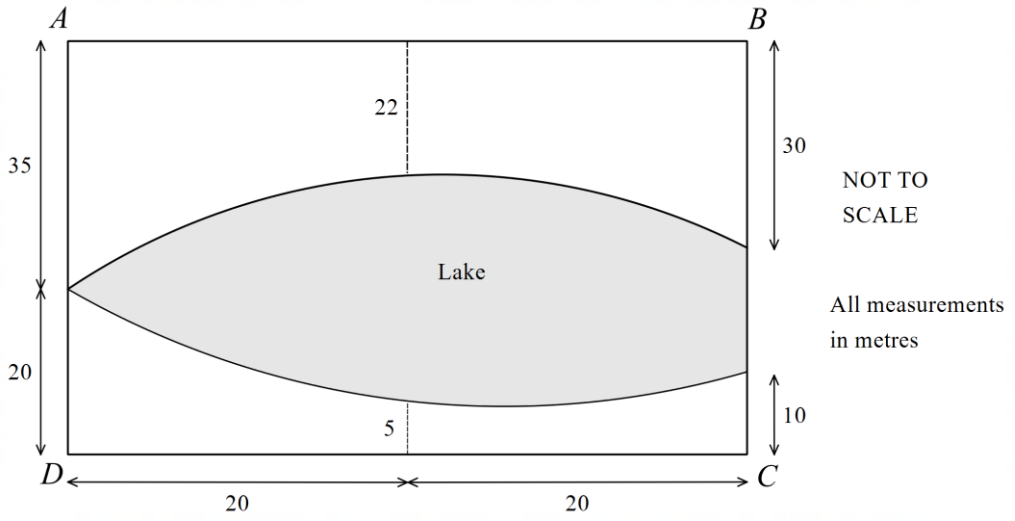
Question 24 (3 marks)

There is a lake enclosed inside a rectangular picnic area $ABCD$ as shown in the diagram.



(a) Use the Trapezoidal rule with three function values to estimate the area of the lake's surface. **2**

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(b) Explain whether the approximation is an overestimate or underestimate of the true area of the lake's surface. **1**

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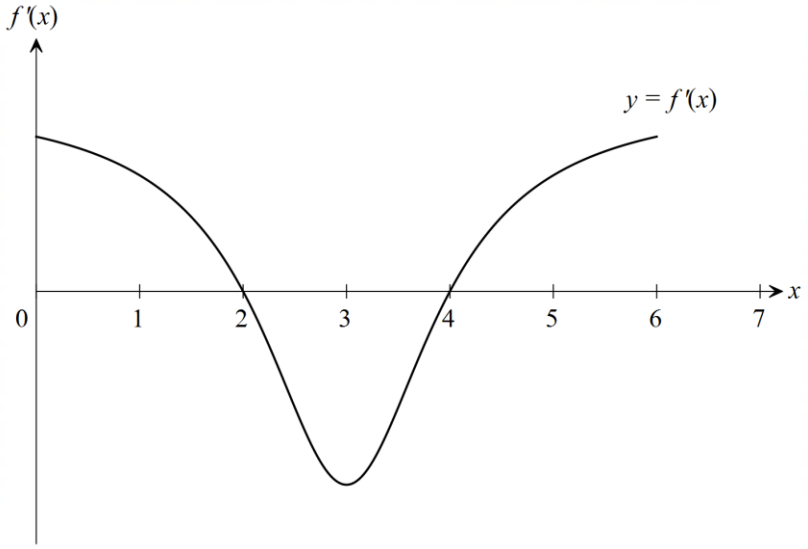
Question 25 (3 marks)

The graph of $f'(x)$ for a certain function $f(x)$ is shown below for the domain $x \in [0,6]$. **3**



It is also known that $f(0) = f(6) = 3$.

In the space below, sketch a possible graph of $y = f(x)$ for $x \in [0,6]$ that incorporates all you can deduce about the function $f(x)$.

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Question 26 (6 marks)

A café is investigating the number of coffees sold during their hours of operation.
The café is open for 8 hours from 7 a.m.

The rate of the number of coffees sold at the café on any given day can be modelled by the function $C(t) = 12 - 10\cos\left(\frac{\pi}{4}t\right)$, where t is the number of hours after 7 am.

(a) Sketch the graph of $y = C(t)$ for $0 \le t \le 8$. **2**

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(b) The café is considered busy when the rate of coffees sold is higher than 17 coffees per hour. Find the times when the café is busy. **2**

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Question 26 continues on page 27

Question 26 (continued)

(c) Find the total number of coffees sold by the café within the 8-hour period on any given day. **2**

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End of Question 26

Question 27 (3 marks)

The wait-time for buying a ticket for a particular concert is normally distributed with a mean of 2 hours and a standard deviation of 0.6 hours.

(a) The concert management plans to give concessions to people who wait for more than 1.4 hours. If 900 people waited to buy a ticket, approximately how many are expected to receive a concession? **2**

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(b) The concert management wants to adjust their quality of service to ensure only 2.5% of wait-times exceed 3 hours. If they wish to maintain the same mean wait-time, what should the new standard deviation be? **1**

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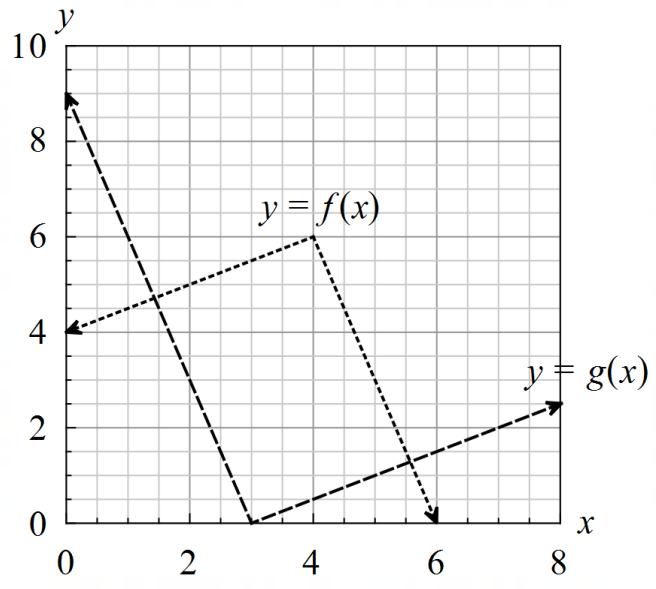
Question 28 (5 marks)

The diagram below shows the graphs of two functions $f(x)$ and $g(x)$.

It is known that $g(x)$ is a transformation of $f(x)$.



(a) List the sequence of transformations applied to $f(x)$ to give $g(x)$. **3**

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(b) Let $u(x) = f(x)g(x)$. Find the value of $u'(1)$. **2**

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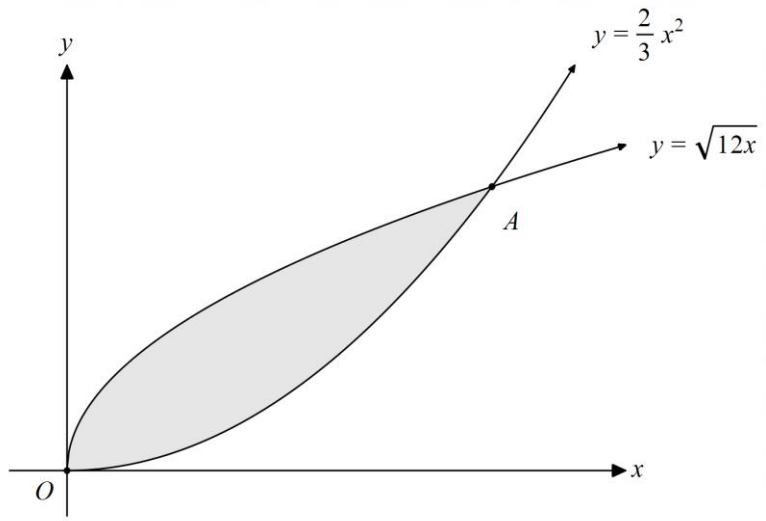
Question 29 (4 marks)

The curves $y = \frac{2}{3}x^2$ and $y = \sqrt{12x}$ intersect at the origin and point A as shown in the diagram below. **4**



Find the shaded area bounded by the two curves.

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End of Section II

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Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 30–34 (29 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 30 (7 marks)

A function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x + 27$.

(a) Find the coordinates and nature of any stationary points and the location of any inflexion points of the graph of $y = f(x)$. **4**

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Question 30 continues on Page 35

Question 30 (continued)

(b) Sketch the graph $y = f(x)$, indicating all stationary points, inflexion points and the y-intercept. **2**

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(c) Hence, state the values of k such that $f(x) = k$ has three solutions. **1**

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End of Question 30

Question 31 (7 marks)

The duration of telemarketing calls to mobile phone users can be modelled as a continuous random variable T minutes, with probability density function:

$$f(t) = \begin{cases} \frac{2}{5}e^{-\frac{2t}{5}} & t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the cumulative distribution function for T. **2**

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(b) Calculate $P(T \leq 8)$. **1**

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Question 31 continues on page 37

Question 31 (continued)

(c) Determine the median call duration. **2**

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(d) A call duration was known to be less than 8 minutes. **2**

Find the probability, that it was longer than 3 minutes.
Give your answer to 2 decimal places.

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End of Question 31

Question 32 (5 marks)

The acceleration of a prototype vehicle is being tested.

Its acceleration, a, is given by $a = \dfrac{20}{(t+1)^2}$ metres per second squared, where time t is measured in seconds.

The vehicle starts from rest and moves in a straight line towards an observation deck 200 metres away.

(a) Find the velocity, v, of the particle at time t. **2**

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(b) The testing finishes after 10 seconds. How far from the observation deck will the vehicle be at this time? Give your answer to two decimal places. **3**

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Question 33 (4 marks)

Lucas and Sophia play a game where they each take turns at throwing two ordinary six-sided dice.

The winner is the first person to throw a double. For example, a double is obtained when the upwards-facing number on both dice shows the number 'two'.

Lucas throws first.

(a) Show that the probability Sophia wins the game on her first or second throw is given by $\frac{5}{36}+\frac{5^3}{6^4}$ **2**

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(b) Calculate the probability that Sophia wins the game. **2**

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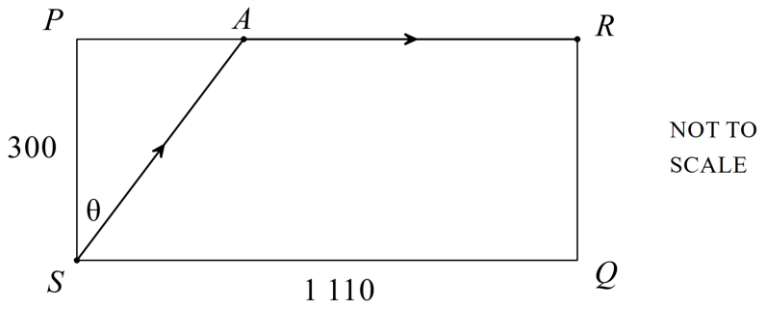
Question 34 (6 marks)

A rectangular field $PRQS$ is surrounded by a running track. The field has dimensions $PS = 300$ metres and $PR = 1110$ metres. A restaurant is located at R.



Sherry is at point S and wants to get to the restaurant in the shortest time possible.
She intends to walk across the field at an angle θ to a point A on the running track, and then run along the track towards R, as shown in the diagram.

Sherry can walk across the field at 4 m/s and run on the track at 6 m/s.

(a) Show that the time, T seconds, Sherry takes to reach the restaurant is given by the equation **2**

$$T = 185 + \frac{75 - 50\sin\theta}{\cos\theta}$$

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Question 34 continues on page 41

Question 34 (continued)

(b) Determine the minimum time required for Sherry to reach the restaurant. **4**

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North Sydney Girls High School

2023 **HSC TRIAL EXAMINATION**

Mathematics Advanced

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Section I – 10 marks (pages 2 – 7)

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- Allow about 15 minutes for this section

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NAME:_____ **TEACHER:**_____

STUDENT NUMBER:

Question	1–10	11–21	22–29	30-34	Total
Mark	/10	/30	/31	/29	/100

Section I

10 marks
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Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 What is the value of $\sqrt{\frac{9.5^2}{e^{-3.8}}}$ to three significant figures?

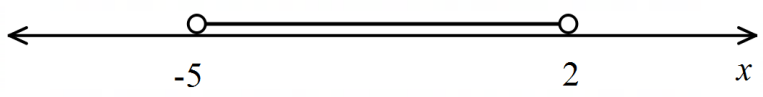
A. 1.42

B. 13.5

C. 63.5

D. 424

2 Which quadratic inequation gives the solution set shown below?



A. $(x-2)(x-5)>0$

B. $(x-2)(x-5)<0$

C. $(x-2)(x+5)>0$

D. $(x-2)(x+5)<0$

3 Let $f(x)=\dfrac{2}{3-x}$ and $g(x)=x-5$. What is the domain of $f(g(x))$?

$f(g(x)) = \dfrac{2}{3-(x-5)}$

$= \dfrac{2}{8-x}$

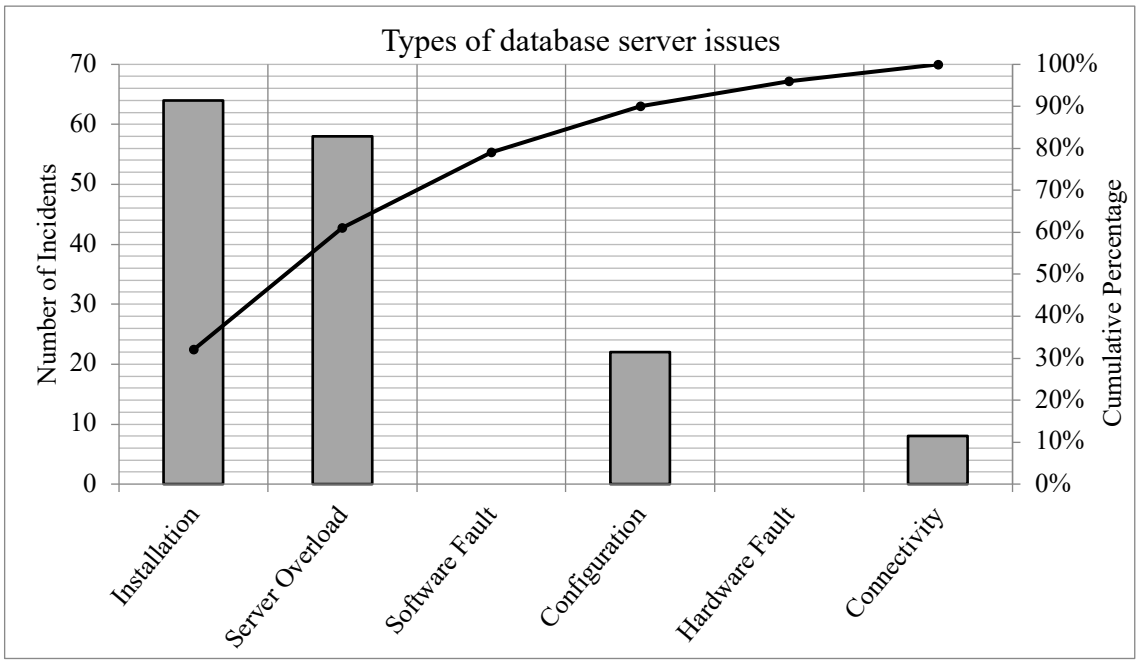
A. $(-\infty,\infty)$

B. $(-\infty,2)\cup(2,\infty)$

C. $(-\infty,3)\cup(3,\infty)$

D. $(-\infty,8)\cup(8,\infty)$

4 The following Pareto chart shows 200 incidents from a report of database server issues. The data for 'Software Fault' and 'Hardware Fault' is missing.



How many incident reports related to the issue of 'Software Fault'?

A. 24 incidents

B. 36 incidents

C. 58 incidents

D. 148 incidents

5 A bus company runs tours around North Sydney. Each tour costs the company $273 in fuel plus $48 per person in other costs. Tourists are charged $65 per person for the tour.

What is the minimum number of tourists the bus company needs to recruit for the tour to break even?

$273+48x-65x=0$

A. 3 tourists

B. 5 tourists

C. 16 tourists

D. 17 tourists

6 A discrete random variable X has the following probability distribution:

x	k	1	3
$P(X=x)$	p	0.4	$2p$

What is the value of k such that $E(X)=0$?

A. -8

B. -3

C. 0

D. 0.3

7 The cohort mean and standard deviation for an assessment task of each course is provided in the table below. The results of a particular student are also listed in the last column.

Course Name	Mean	Standard Deviation	Student Mark
English	71	6.3	76
Mathematics	68	5.8	72
Legal Studies	32	1.4	34
Chemistry	24	2.7	22

Which subject did the student perform better in compared to their respective means?

A. English

B. Mathematics

C. Legal Studies

D. Chemistry

8 What is the primitive function of $x+\sec^2 x$?

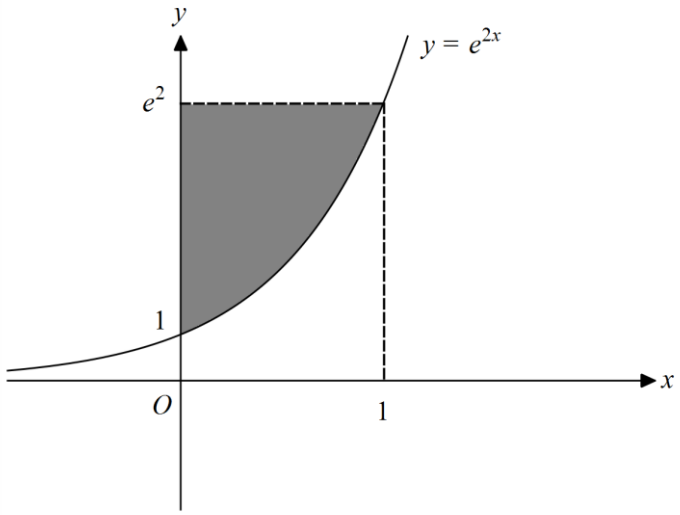
A. $\frac{x^2}{2}+\frac{1}{3}\sec^3(x)+c$

B. $x+\frac{1}{2}\sec(x)+c$

C. $\frac{x^2}{2}+\tan(x)+c$

D. $\frac{x^2}{2}+\frac{1}{2}\tan(x)+c$

9 Four different students proposed the following calculations to find the area of the shaded region in the diagram below.



Student 1: $\int_0^1 e^{2x}\,dx$

Student 2: $e^2 - \int_0^1 e^{2x}\,dx$

Student 3: $\int_1^{e^2} e^{2y}\,dx$

Student 4: $\int_0^{e^2} \frac{\log_e y}{2}\,dy$

Which student(s) is correct?

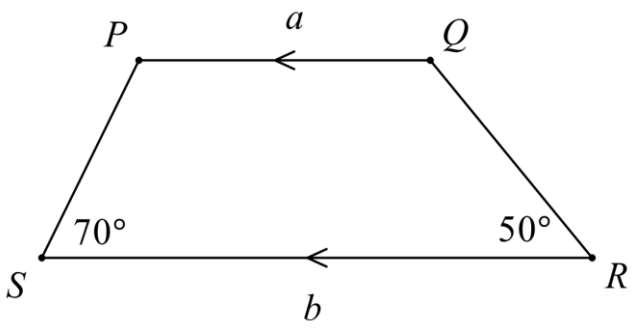
A. Student 2 only

B. Students 2 and 3 only

C. Students 1 and 4 only

D. Students 2 and 4 only

10 In the figure below, $PQ \parallel RS$, $PQ = a$ and $RS = b$. What is the length of QR?



$\frac{\sin 70}{QR} = \frac{\sin 60^\circ}{b-a}$

A. $\dfrac{(b+a)\sin 70^\circ}{\sin 60^\circ}$

B. $\dfrac{(b+a)\sin 70^\circ}{\sin 50^\circ}$

C. $\dfrac{(b-a)\sin 70^\circ}{\sin 60^\circ}$

D. $\dfrac{(b-a)\sin 70^\circ}{\sin 50^\circ}$

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Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–34
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–21 (30 marks)
Booklet 2 – Attempt Questions 22–29 (31 marks)
Booklet 3 – Attempt Questions 30–34 (29 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

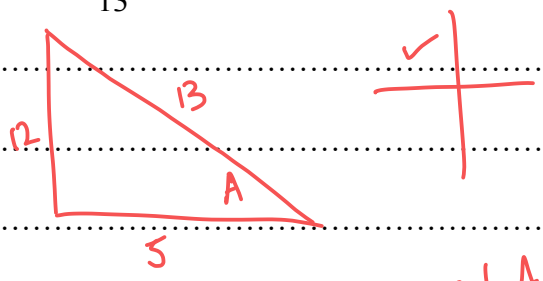
Solve $|5-x|=3$. **2**

$5-x=3$ or $5-x=-3$

$x=2$ $x=8$

Question 12 (2 marks)

If $\sin A=\frac{12}{13}$ and A is obtuse, find the exact value of $\cot A$. **2**



$\cot A=\frac{-5}{12}$

Question 13 (2 marks)

Find the values of m if the equation $4x^2+(m+3)x+9=0$ has two equal roots. **2**

$\Delta=(m+3)^2-4\times9\times4=0$ for equal roots

$(m+3)^2=144$

$m+3=\pm12$

$m=-15, 9$

Question 14 (3 marks)

It is known that the fourth and ninth terms of an arithmetic sequence are 48 and 1925 respectively.

(a) Find the common difference and the first term of the sequence. **2**

$T_4 = a + 3d = 48$

$T_9 = a + 8d = 1925$

$5d = 1877$

$d = \frac{1877}{5} = 375.4$

$\therefore a = -1078.2$

(b) Find the sum of the first 10 terms of the sequence. **1**

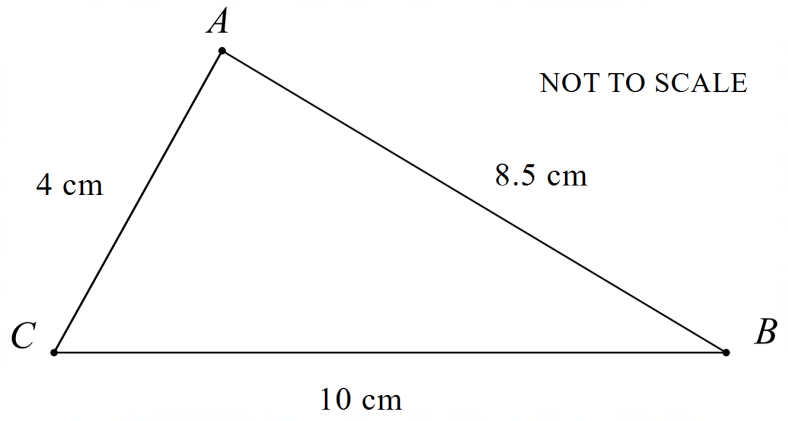
$S_{10} = \frac{10}{2}(2 \times (-1078.2) + 9 \times 375.4)$

$= 5(-2156.4 + 3378.6)$

$= 5 \times 1222.2$

$= 6111$

Question 15 (2 marks)



In the diagram above, ABC is a triangle with $AB = 8.5$ cm, $AC = 4$ cm and $BC = 10$ cm. **2**

Determine the size of $\angle ACB$, correct to the nearest degree.

$AB^2 = AC^2 + BC^2 - 2BC \times AC \cos \angle ACB$

$8.5^2 = 4^2 + 10^2 - 2 \times 4 \times 10 \cos \angle ACB$

$\cos \angle ACB = \frac{16 + 100 - 72.25}{80}$

$= \frac{43.75}{80}$

$\therefore \angle ACB = 57^\circ$ (nearest degree)

Question 16 (3 marks)

Find the equation of the tangent to the curve $y = x^2e^{3x}$ at the point where $x = \frac{1}{3}$. **3**

When $x = \frac{1}{3}$

$y = \left(\frac{1}{3}\right)^2 e$

$= \frac{e}{9}$

$\therefore$ Point at $\left(\frac{1}{3}, \frac{e}{9}\right)$

$y' = 2xe^{3x} + 3x^2e^{3x}$

When $x = \frac{1}{3}$

$m_T = \frac{2}{3}e + \frac{1}{3}e = e$

Equation: $y - \frac{e}{9} = e\left(x - \frac{1}{3}\right)$

$\therefore 9y - e = 9ex - 3e$

$\therefore 9ex - 9y - 2e = 0$ is equation of tangent

Question 17 (3 marks)

Show that $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$. **3**

$$\text{LHS} = \frac{(1+\sin x)(1+\sin x)+\cos x(\cos x)}{\cos x(1+\sin x)}$$

$$= \frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$$

$$= \frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$= \frac{2(1+\sin x)}{\cos x(1+\sin x)}$$

$$= 2\sec x$$

$$= \text{RHS}$$

Question 18 (4 marks)

The number of birds, N, in a colony at time t years after observation begins is represented by the model $N = 100 + Ae^{-0.1t}$. Initially, there were 400 birds in the colony.

(a) Find the value of A. **1**

when $t = 0$, $N = 400$

$400 = 100 + A$

$A = 300$

(b) The colony is considered inactive when the number of birds fall below 125. **2**

After how many years will the colony be inactive? Give your answer to 2 decimal places.

$N = 100 + 300e^{-0.1t}$

when $N = 125$

$125 = 100 + 300e^{-0.1t}$

$\frac{25}{300} = e^{-0.1t}$

$-0.1t = \ln\left(\frac{1}{12}\right)$

$t = -10\ln(12)$

$= 24.85$ years (2dp)

Colony inactive after 24.85 years

(c) What is the eventual population of the colony? **1**

As $t \to \infty$

$N \to 100$

Question 19 (3 marks)

Solve $2\sin^2 x - \sin x = 1$ for $x \in [0, 2\pi]$ **3**

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = \frac{-1}{2}$ $\sin x = 1$

$x = \frac{\pi}{2}$

Reference: $\frac{\pi}{6}$

$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

Question 20 (2 marks)

Find $\int x^2 2^{x^3}\, dx$. **2**

$= \frac{1}{3}\int 3x^2 2^{x^3}\, dx$

$= \frac{\frac{1}{3}(2^{x^3})}{\ln(2)} + C$

$= \frac{2^{x^3}}{3\ln 2} + C$

Question 21 (4 marks)

Let $y = \ln\left(\frac{4-x}{4+x}\right)$

(a) Show that $\frac{dy}{dx} = \frac{8}{x^2 - 16}$ **2**

$y = \ln(4-x) - \ln(4+x)$

$\frac{dy}{dx} = \frac{-1}{4-x} - \frac{1}{4+x}$

$= \frac{-4-x-4+x}{(4+x)(4-x)}$

$= \frac{-8}{16-x^2}$

$= \frac{8}{x^2-16}$

(b) Hence, evaluate $\int_{-3}^{3} \frac{24}{x^2 - 16}\, dx$, giving your answer as an exact value. **2**

$\int_{-3}^{3} \frac{24}{x^2-16}\, dx$

$= 3\int_{-3}^{3} \frac{8}{x^2-16}\, dx$

$= 3\left[\ln\left(\frac{4-x}{4+x}\right)\right]_{-3}^{3}$

$= 3\ln\left(\frac{1}{7}\right) - 3\ln\left(\frac{7}{1}\right)$

$= 3\ln\left(\frac{1}{49}\right) = -3\ln(49)$

End of Section I

Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 22–29 (31 marks)

Instructions

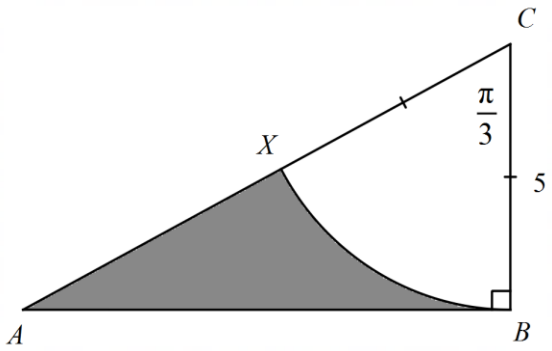
- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 22 (3 marks)

In ΔABC, $\angle ABC = \frac{\pi}{2}$, $\angle ACB = \frac{\pi}{3}$ and $BC = 5$ units. **3**

A circular arc, of centre C and radius CB, cuts the side CA at X.



Find the exact area of the shaded region ABX.

$A_{sector} = \frac{1}{2} \times 5^2 \times \frac{\pi}{3}$

$= \frac{25\pi}{6}$

In ΔABC, $\tan\frac{\pi}{3} = \frac{AB}{5}$

$AB = 5\tan\left(\frac{\pi}{3}\right)$

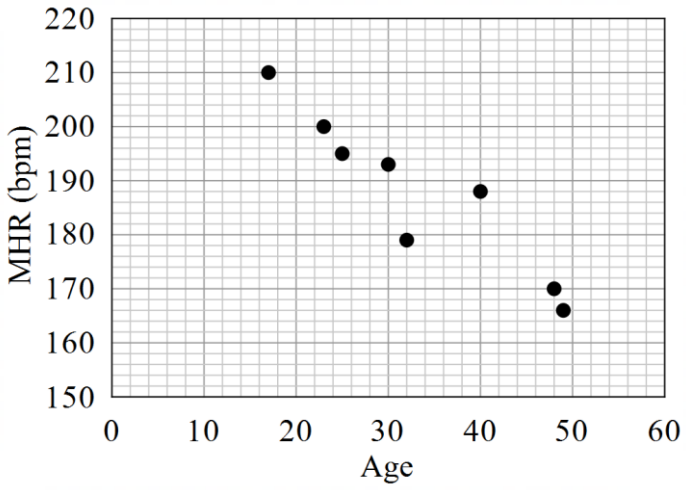
Area $\Delta ABC = \frac{25\sqrt{3}}{2}$

$\therefore A_{shaded} = \left(\frac{25\sqrt{3}}{2} - \frac{25\pi}{6}\right) u^2$

Question 23 (4 marks)

An anonymous survey was conducted at a local gym, which collected information about the respondents' age and perceived maximum heart rate (MHR) measured in beats per minute (bpm). A random sample of eight responses were selected and shown in the table and scatterplot below.

Response	A	B	C	D	E	F	G	H
Age	17	23	25	30	32	40	48	49
MHR (bpm)	210	200	195	193	180	185	170	169



(a) Find the correlation coefficient and describe the association between the age and the maximum heart rate. **2**

$r = -0.95$

There is a very strong, negative, linear relationship between age and maximum heart rate

(b) Find the equation of the least squares regression line using your calculator. **2**
Use the equation to predict the MHR of a 36-year old respondent.

$MHR = 226.59 - 1.18 \times Age$

when age = 36

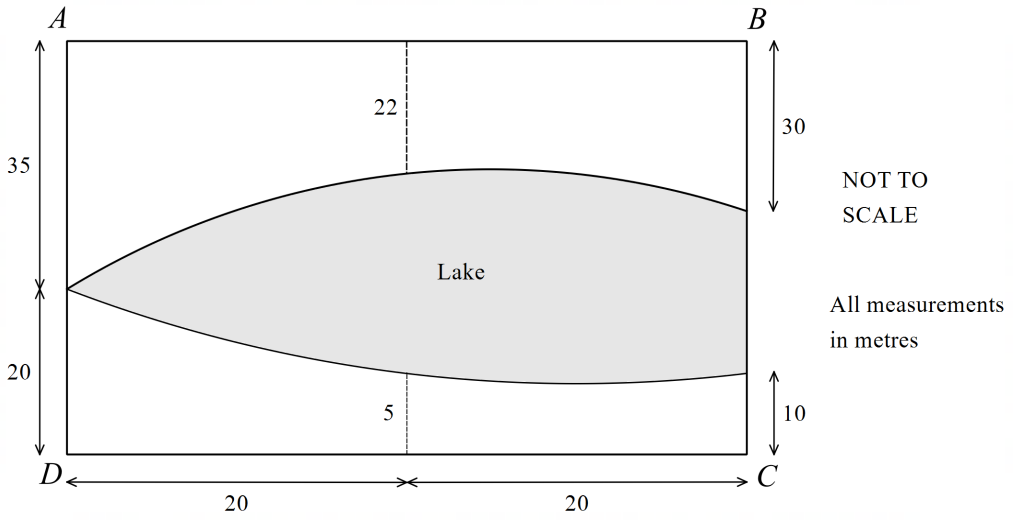
$MHR = 226.59 - 1.18 \times 36$

$= 184.11$ bpm

∴ The predicted MHR for a 36 year old respondent is 184 bpm.

Question 24 (3 marks)

There is a lake enclosed inside a rectangular picnic area $ABCD$ as shown in the diagram.



(a) Use the Trapezoidal rule with three function values to estimate the area of the lake's surface. **2**

$h = \frac{40-0}{2} = 20$

x	0	20	40
width	0	28	15

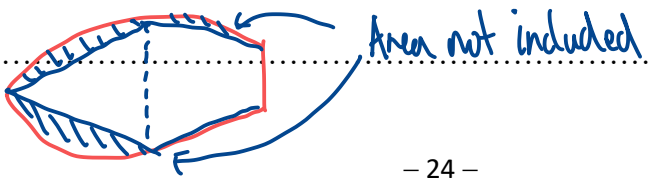
$A \approx \left(\frac{40-0}{2}\right)(0+15+2\times 28)$

$= 10 \times 71$

$= 710 m^2$

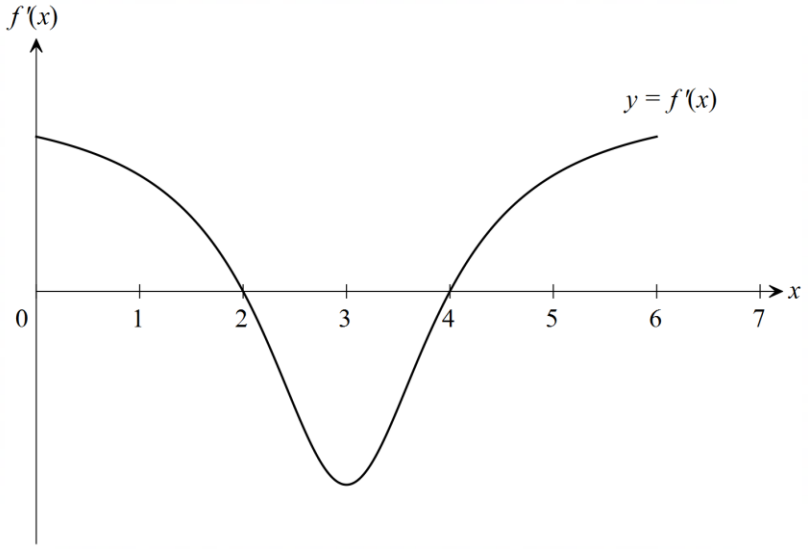
(b) Explain whether the approximation is an overestimate or underestimate of the true area of the lake's surface. **1**

The Trapezoidal Rule will underestimate the area of the lake as the boundaries of the lake curve outwards towards the edge of the park. Therefore, the trapeziums created will exclude the areas created by the lake and secants (see diagram below).



Question 25 (3 marks)

The graph of $f'(x)$ for a certain function $f(x)$ is shown below for the domain $x \in [0,6]$. **3**



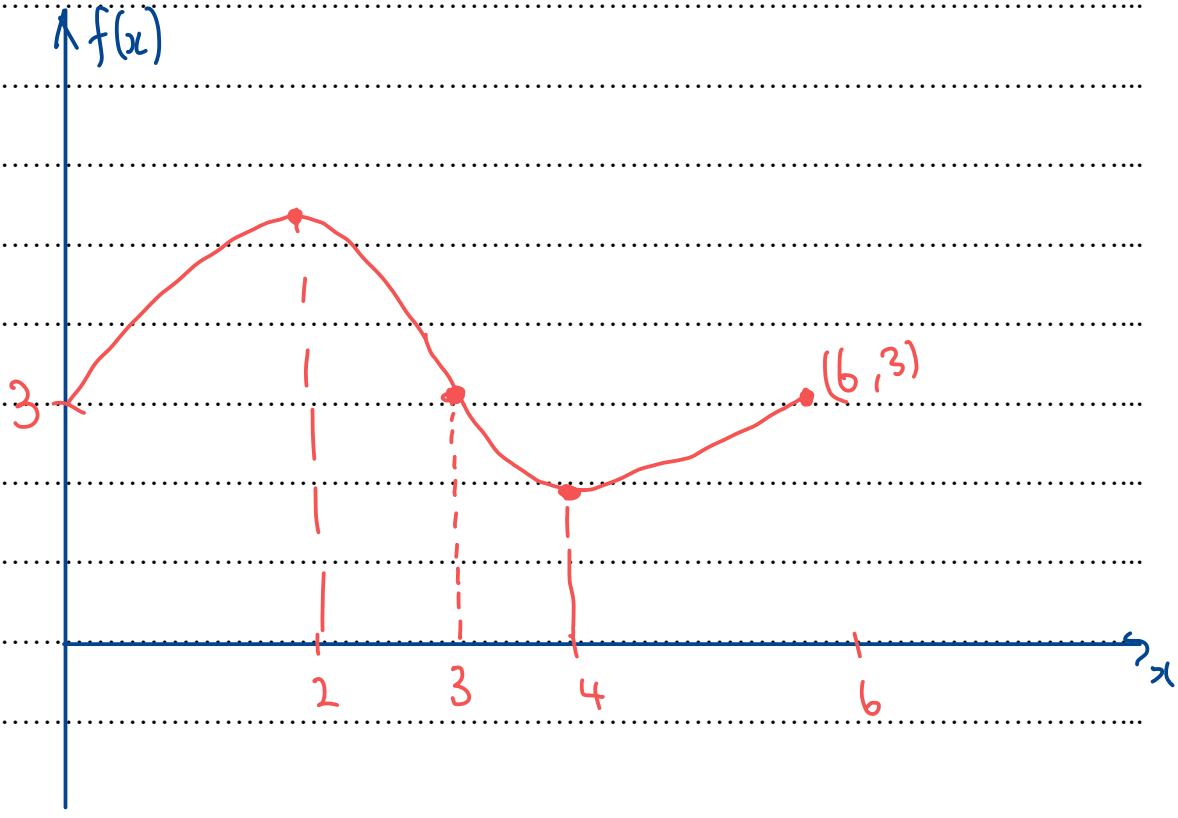
It is also known that $f(0) = f(6) = 3$.

In the space below, sketch a possible graph of $y = f(x)$ for $x \in [0,6]$ that incorporates all you can deduce about the function $f(x)$.

x	0	2	3	4	5
$f'(x)$	+	0	−	0	+

Max at $x=2$

Min at $x=4$

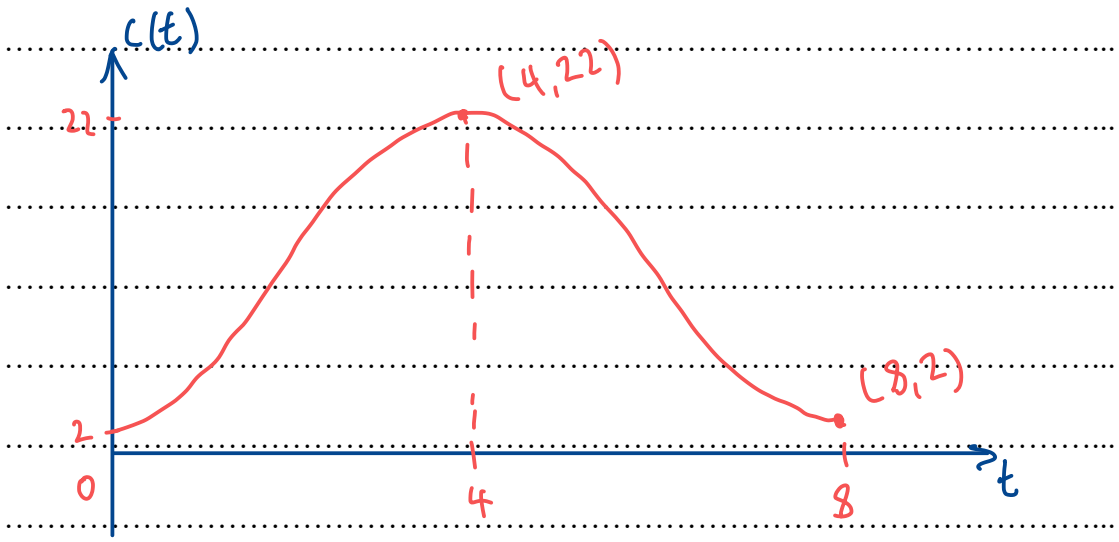


Question 26 (6 marks)

A café is investigating the number of coffees sold during their hours of operation.
The café is open for 8 hours from 7 a.m.

The rate of the number of coffees sold at the café on any given day can be modelled by the function $C(t) = 12 - 10\cos\left(\frac{\pi}{4}t\right)$, where t is the number of hours after 7 am.

(a) Sketch the graph of $y = C(t)$ for $0 \le t \le 8$. **2**



(b) The café is considered busy when the rate of coffees sold is higher than 17 coffees per hour. Find the times when the café is busy. **2**

When $C(t) = 17$

$17 = 12 - 10\cos\left(\frac{\pi}{4}t\right)$

$\cos\left(\frac{\pi}{4}t\right) = -\frac{1}{2}$

$\frac{\pi}{4}t = \frac{2\pi}{3}, \frac{4\pi}{3}$

$t = \frac{8}{3}, \frac{16}{3}$ = 2 hrs 40 min or 5 hrs 20 min

∴ Cafe is busy from 9:40 am to 12:20 pm

Question 26 continues on page 27

(c) Find the total number of coffees sold by the café within the 8-hour period on any given day. **2**

Total sold $= \int_0^8 C(t)\,dt$

$= \int_0^8 12 - 10\cos\left(\frac{\pi}{4}t\right)\,dt$

$= \left[12t - \frac{40}{\pi}\sin\left(\frac{\pi}{4}t\right)\right]_0^8$

$= \left[12\times 8 - \frac{40}{\pi}\sin\left(\frac{\pi}{4}\times 8\right)\right] - (0-0)$

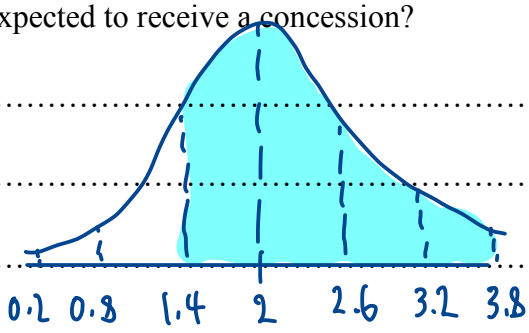
$= 96$ coffees

End of Question 26

Question 27 (3 marks)

The wait-time for buying a ticket for a particular concert is normally distributed with mean of 2 hours and standard deviation of 0.6 hours.

(a) The concert management plans to give concessions to people who wait for more than 1.4 hours. If 900 people waited to buy a ticket, approximately how many are expected to receive a concession? **2**



Let X be the wait-time

$P(X \geq 1.4)$

Using empirical values, this represents $\left(50 + \frac{68}{2}\right)\%$

$= 84\%$

Expected number of people with concessions is

$900 \times 84\% = 756$ people

(b) The concert management wants to adjust their quality of service to ensure only 2.5% of wait-times exceed 3 hours. If they wish to maintain the same mean wait-time, what should the new standard deviation be? **1**

$\bar{x} = 2$

$\sigma = ?$

For $P(X \geq 3) = 0.025$, the z-score is 2 using empirical values

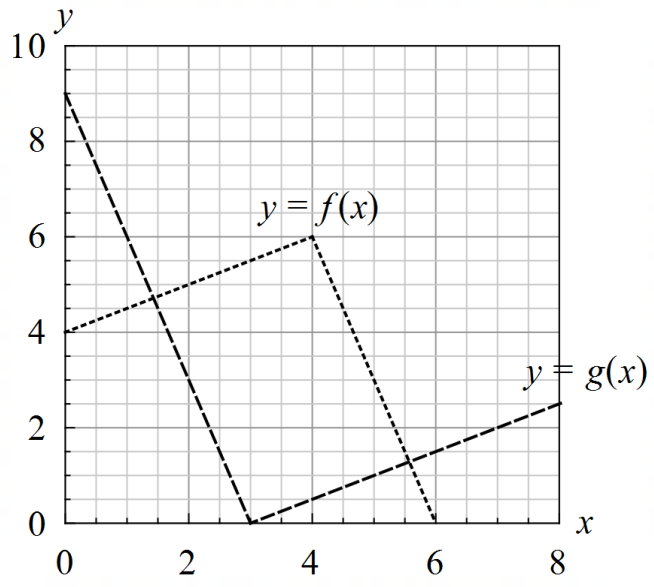
$\therefore 2 = \frac{3-2}{\sigma}$

$\sigma = \frac{1}{2}$

Question 28 (5 marks)

The diagram below shows the graphs of two functions $f(x)$ and $g(x)$.

It is known that $g(x)$ is a transformation of $f(x)$.



(a) List the sequence of transformations applied to $f(x)$ to give $g(x)$. **3**

(1) Horizontal reflection about y-axis

(2) Horizontal shift right by 7 units

(3) Vertical reflection about x-axis

(4) Vertical shift up by 6 units

(b) Let $u(x) = f(x)g(x)$. Find the value of $u'(1)$. **2**

$u'(x) = f'(x)g(x) + f(x)g'(x)$

$u'(1) = f'(1)g(1) + f(1)g'(1)$

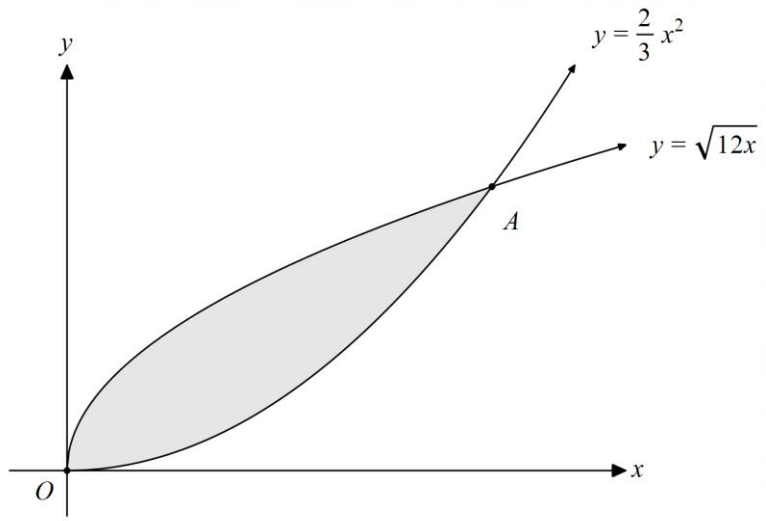
$= \frac{1}{2} \times 6 + 4.5 \times -3$

$= 3 - 13.5$

$= -10.5$

Question 29 (4 marks)

The curves $y = \frac{2}{3}x^2$ and $y = \sqrt{12x}$ intersect at the origin and point A as shown in the diagram below. **4**



Find the shaded area bounded by the two curves.

Intersection: $\frac{2}{3}x^2 = \sqrt{12x}$

$4x^4 = 108x$

$4x^4 - 108x = 0$

$4x(x^3 - 27) = 0$

$x = 0, \quad x^3 = 27$

$x = 3$

Area: $A = \int_0^3 (12x)^{\frac{1}{2}} - \frac{2}{3}x^2 \, dx$

$= \left[\frac{2}{3}\sqrt{12}\, x^{\frac{3}{2}} - \frac{2}{9}x^3\right]_0^3$

$= \frac{2}{3}\sqrt{12} \times 3^{\frac{3}{2}} - \frac{2}{9} \times 27 - (0 - 0)$

$= \frac{2}{3} \times 3 \times \sqrt{36} - 6$

$= 6 \text{ units}^2$

End of Section II

Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 30–34 (29 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 30 (7 marks)

A function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x + 27$.

(a) Find the coordinates and nature of any stationary points and the location of any inflexion points of the graph of $y = f(x)$. **4**

$f'(x) = 3x^2 - 6x - 9$

For stationary points, $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

$y = 27 - 27 - 27 + 27$ $\quad$ $y = -1 - 3 + 9 + 27$

$= 0$ $\quad$ $= 32$

$\therefore$ Stationary points at $(3, 0)$ and $(-1, 32)$

$f''(x) = 6x - 6$

$f''(3) = 12 > 0$ $\quad$ $f''(-1) = -12 < 0$

$\therefore$ MIN at $(3, 0)$ $\quad$ $\therefore$ MAX at $(-1, 32)$

For inflexion points, $f''(x) = 0$

$6x - 6 = 0$

$x = 1$

$y = 16$

x	0	1	2
$f''(x)$	-6	0	6
	$\cap$	$-$	$\cup$

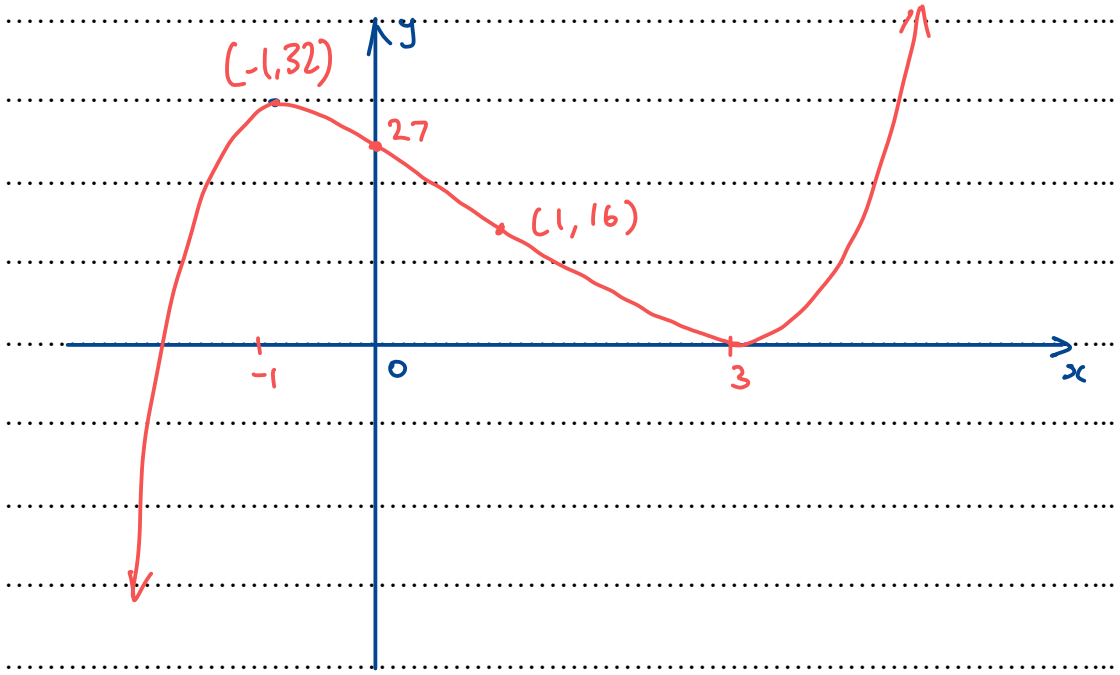
Concavity changes at $(1, 16)$

$\therefore$ Inflexion point at $(1, 16)$

Question 30 continues on Page 35

Question 30 (continued)

(b) Sketch the graph $y = f(x)$, indicating all stationary points, inflexion points and the y-intercept. **2**



(c) Hence, state the values of k such that $f(x) = k$ has three solutions. **1**

$0 < k < 32$

End of Question 30

Question 31 (7 marks)

The duration of telemarketing calls to mobile phone users can be modelled as a continuous random variable T minutes, with probability density function:

$$f(t)=\begin{cases} \frac{2}{5}e^{-\frac{2t}{5}} & t\geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the cumulative distribution function for T. **2**

$F(t)=\int_0^t \frac{2}{5}e^{\frac{-2x}{5}}\,dx$

$=\left[\frac{2}{5}\times\frac{-5}{2}e^{\frac{-2x}{5}}\right]_0^t$

$=\left[-e^{\frac{-2x}{5}}\right]_0^t$

$=1-e^{-\frac{2}{5}t}$

(b) Calculate $P(T\leq 8)$. **1**

$F(8)=P(T\leq 8)$

$=1-e^{\frac{-16}{5}}$

$=0.959$ (3dp)

Question 31 continues on page 37

Question 31 (continued)

(c) Determine the median call duration. **2**

We want $F(t) = 0.5$

$1 - e^{-0.4t} = 0.5$

$e^{-0.4t} = 0.5$

$-0.4t = \ln(0.5)$

$t = \frac{5}{2}\ln(2)$ minutes

Median call duration is approximately 1.73 minutes

(d) A call duration was known to be less than 8 minutes. **2**

Find the probability, that it was longer than 3 minutes.
Give your answer to 2 decimal places.

$$P(T \geq 3 \mid T \leq 8) = \frac{P(T \geq 3 \cap T \leq 8)}{P(T \leq 8)}$$

$$= \frac{P(3 \leq T \leq 8)}{P(T \leq 8)}$$

$$= \frac{F(8) - F(3)}{F(8)}$$

$$= \frac{1 - e^{-\frac{16}{5}} - (1 - e^{-\frac{6}{5}})}{1 - e^{-\frac{16}{5}}}$$

$$= \frac{e^{-\frac{6}{5}} - e^{-\frac{16}{5}}}{1 - e^{-\frac{16}{5}}}$$

$$\approx 0.27 \ (2\text{dp})$$

End of Question 31

Question 32 (5 marks)

The acceleration of a prototype vehicle is being tested.

Its acceleration, a, is given by $a = \dfrac{20}{(t+1)^2}$ metres per second squared, where time t is measured in seconds.

The vehicle starts from rest and moves in a straight line towards an observation deck 200 metres away.

(a) Find the velocity, v, of the particle at time t. **2**

$v = \int \frac{20}{(t+1)^2}\,dt$

$= \frac{-20}{t+1} + c$

when $t=0$, $v=0$ $\therefore c = 20$

$\therefore v = \frac{-20}{t+1} + 20$

(b) The testing finishes after 10 seconds. How far from the observation deck will the vehicle be at this time? Give your answer to two decimal places. **3**

$x = -20\ln|t+1| + 20t + c$

when $t=0$, $x=0$

$\therefore c = 0$

$\therefore x = 20t - 20\ln|t+1|$

when $t = 10$

$x = 200 - 20\ln(11)$

$= 152.04$ And $200 - 152.04 = 47.96$

$\therefore$ Vehicle is 47.96m away from observation deck

Question 33 (5 marks)

Lucas and Sophia play a game where they each take turns at throwing two ordinary six-sided dice.

The winner is the first person to throw a double. For example, a double is obtained when the upwards-facing number on both dice shows the number 'two'.

Lucas throws first.

(a) Show that the probability Sophia wins the game on her first or second throw is given by $\frac{5}{36}+\frac{5^3}{6^4}$ **2**

$P(\text{double}) = \frac{6}{36} = \frac{1}{6}$ } for any player

$\therefore P(\text{no double}) = 1-\frac{1}{6} = \frac{5}{6}$ }

For Sophia to win first, Lucas loses then Sophia wins

$P(S \text{ wins first}) = \frac{5}{6} \times \frac{1}{6}$

$= \frac{5}{36}$

For Sophia to win on second turn, each lose once then Lucas loses

$P(S \text{ wins on } 2^{nd}) = \left(\frac{5}{6} \times \frac{5}{6}\right) \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{5^3}{6^4}$

$\therefore$ Sophia wins on first or second turn is $\frac{5}{36} + \frac{5^3}{6^4}$

(b) Calculate the probability that Sophia wins the game. **2**

$P(\text{Sophia wins}) = P(S \text{ wins on 1st}) + P(S \text{ wins on } 2^{nd}) + \ldots$

$= \frac{5}{36} + \frac{5^3}{6^4} + \frac{5^5}{6^6} + \ldots$

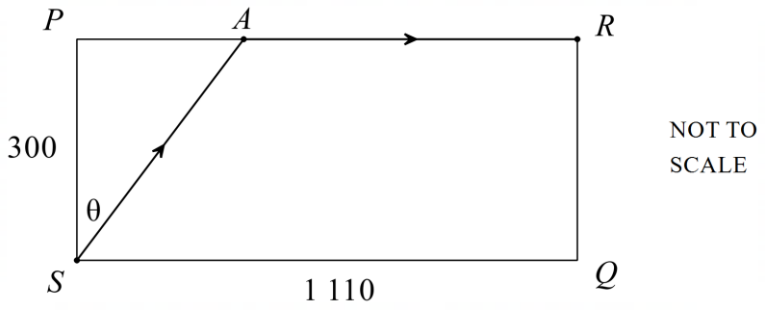
which forms an infinite GP with $a = \frac{5}{36}$, $r = \frac{5^2}{6^2}$

$\therefore P(S \text{ wins}) = \frac{\frac{5}{36}}{1-\frac{5^2}{6^2}} = \frac{5}{36-25}$

$= \frac{5}{11}$

Question 34 (6 marks)

A rectangular field $PRQS$ is surrounded by a running track. The field has dimensions $PS = 300$ metres and $PR = 1110$ metres. A restaurant is located at R.



Sherry is at point S and wants to get to the restaurant in the shortest time possible.
She intends to walk across the field at an angle θ to a point A on the running track, and then run along the track towards R.

Sherry can walk across the field at 4 m/s and run on the track at 6 m/s.

(a) Show that the time, T seconds, Sherry takes to reach the restaurant is given by the equation **2**

$$T = 185 + \frac{75 - 50\sin\theta}{\cos\theta}$$

In $\triangle APS$

$\cos\theta = \frac{PS}{AS}$ $\tan\theta = \frac{PA}{PS}$

$\therefore AS = \frac{300}{\cos\theta}$ $\therefore PA = 300\tan\theta$

$\therefore AR = 1110 - 300\tan\theta$

Time taken:

$T_{AS} = \frac{\frac{300}{\cos\theta}}{4}$ $T_{AR} = \frac{1110 - 300\tan\theta}{6}$

$= \frac{75}{\cos\theta}$ $= 185 - 50\tan\theta$

$\therefore$ Total time taken: $T = \frac{75}{\cos\theta} + 185 - \frac{50\sin\theta}{\cos\theta}$

$T = 185 + \frac{75 - 50\sin\theta}{\cos\theta}$

Question 34 continues on page 41

Question 34 (continued)

(b) Determine the minimum time required for Sherry to reach the restaurant. **4**

$$\frac{dT}{d\theta} = \frac{(-50\cos\theta)(\cos\theta) - (-\sin\theta)(75 - 50\sin\theta)}{\cos^2\theta}$$

$$= \frac{-50\cos^2\theta - 50\sin^2\theta + 75\sin\theta}{\cos^2\theta}$$

$$= \frac{-50 + 75\sin\theta}{\cos^2\theta}$$

For stationary points, $\frac{dT}{d\theta} = 0$

$\therefore -50 + 75\sin\theta = 0$

$\sin\theta = \frac{2}{3}$

$\theta = 0.729\ldots$, $2.411\ldots$ (discard) since $0 < \theta < \frac{\pi}{2}$

θ	0.1	0.729	1
$\frac{dT}{d\theta}$	-42.9	0	44.9
	\	–	/

$\therefore$ Minimum when $\theta = 0.729$ rad.

$\therefore T_{min} = 185 + \frac{75 - 50\sin(0.729)}{\cos(0.729)}$

$= 240.9$ seconds

$\therefore$ Minimum time taken is approximately 241 seconds

End of paper